

Module-3

Induction Motor Drives

Syllabus:

Induction Motor Drives:

- Analysis and Performance of Three Phase Induction Motors,
- Operation with Unbalanced Source Voltage and Single Phasing
- Operation with Unbalanced Rotor Impedances
- Analysis of Induction Motor Fed From Non-Sinusoidal Voltage Supply
- Starting, Braking, Transient Analysis.
- Speed Control Techniques-Stator Voltage Control
- Variable Voltage Frequency Control from Voltage Sources.

Introduction

Induction Motor: The motor which runs at a speed less than synchronous speed, due to slip which is the difference in the stator field and rotor field

Induction motors, particularly squirrel cage IM, have many advantages when compared to DC motors. They are,

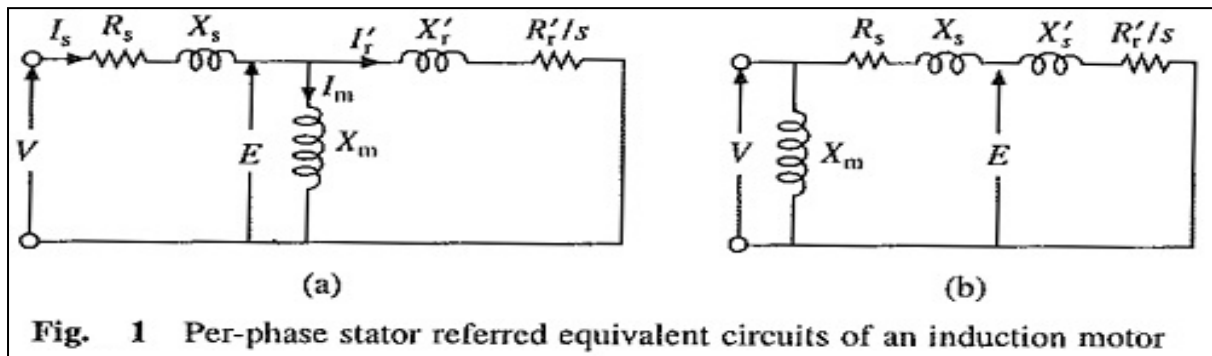
- Ruggedness
- Lower maintenance requirements
- Better reliability
- Low cost, less weight and volume
- Higher efficiency
- Also induction motors are able to operate in dirty and explosive environments.

Three Phase Induction Motor

Three Phase Induction Motors are of two types:

1. **Squirrel-case IM**-In squirrel-cage, the rotor consists of longitudinal conductor-bars shorted by circular connectors at the two ends
 2. **Wound-rotor IM**- In wound-rotor motor, the rotor also has a balanced three-phase distributed winding having same poles as stator winding
- However, in both, stator carries a three-phase balanced distributed winding..

Analysis & Performance of Three Phase Induction Motors



- Per-phase equivalent circuit of a Three Phase Induction Motors is shown in Fig(a), simplified equivalent circuit is shown in Fig(b).
- R'_r and X'_r - stator referred values of rotor resistance R_r and rotor reactance X_r .
- Slip is defined by

$$s = \frac{\omega_{ms} - \omega_m}{\omega_{ms}} \quad (1) \quad \text{where } \omega_m \text{ and } \omega_{ms} \text{ are rotor and synchronous speeds, respectively}$$

$$\omega_{ms} = \frac{4\pi f}{p} \text{ rad/sec} \quad (2) \quad \text{where } f \text{ and } p \text{ are supply frequency and number of poles, respectively.}$$

From Eq(1): $\omega_m = \omega_{ms}(1 - s) \quad (3)$

Rotor current referred to stator side :

$$\bar{I}'_r = \frac{V}{\left(R_s + \frac{R'_r}{s}\right) + j(X_s + X'_r)} \quad (4)$$

Power transferred to rotor (or air-gap power)

$$P_g = 3I'^2_r R'_r / s \quad (5)$$

Rotor copper loss is

$$P_{cu} = 3I'^2_r R'_r \quad (6)$$

Electrical power converted into mechanical power

$$P_m = P_g - p_{cu} = 3I'^2_r R'_r \left(\frac{1-s}{s}\right) \quad (7)$$

Torque developed by motor

$$T = P_m / \omega_m \quad (8)$$

Substitute for P_m and ω_m :

$$T = \frac{3}{\omega_{ms}} I_r'^2 \frac{R_r'}{s} \quad (9)$$

Substitute for I_r'

$$T = \frac{3}{\omega_{ms}} \left[\frac{V^2 R_r' / s}{\left(R_s + \frac{R_r'}{s} \right)^2 + (X_s + X_r')^2} \right] \quad (10)$$

Comparing eq. 5 and eq.9

$$T = P_g / \omega_{ms} \quad (11)$$

Differentiating T in eq.10, with respect to s and equating to zero gives the slip for maximum torque

$$s_m = \pm \frac{R_r'}{\sqrt{R_s^2 + (X_s + X_r')^2}}$$

+ve: Working as IM

-ve: Working as IG

(12)

Substitute above s_m in Torque eq.10 to get T_{max}

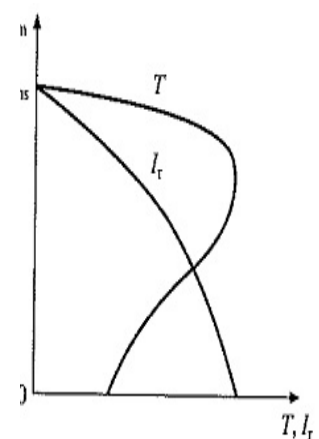
$$T_{max} = \frac{3}{2\omega_{ms}} \left[\frac{V^2}{R_s \pm \sqrt{R_s^2 + (X_s + X_r')^2}} \right]$$

(13)

Maximum torque is also known as breakdown torque. While it is independent of rotor resistance, s_m is directly proportional to rotor resistance.

The natures of speed-torque and speed-rotor current characteristics are shown in Fig.2.

- Both rotor-current and torque are zero at synchronous speed.
- With decrease in speed, both increase.
- While torque reduces after reaching breakdown value, the rotor-current continues to increase, reaching a maximum value at zero speed.



Speed torque and speed rotor current

- Dividing Eq. (10) by (13) and then substituting from (12) yields

$$\frac{T}{T_{\max}} = \frac{2 \left(1 + \frac{R_s}{R_r'} s_m \right)}{\frac{s}{s_m} + \frac{s_m}{s} + 2 \frac{R_s}{R_r' s_m}} \quad (14)$$

- For slips much smaller than s_m , second term of the denominator dominates.
- Therefore, speed-torque relation from 0 to rated torque is approximately represented by a straight line.
- For slips much larger than s_m , first term of the denominator dominates and speed-torque relation takes a hyperbolic shape in this region.
- In the whole region of motor operation, term $(R_s s_m / R_r')$ is small compared to 1 and dominating term in the denominator. Therefore, it can be dropped from Eq. (14). Thus

$$\frac{T}{T_{\max}} = \frac{2}{\frac{s}{s_m} + \frac{s_m}{s}} \quad (15)$$

Induction Motors with Special Designs:

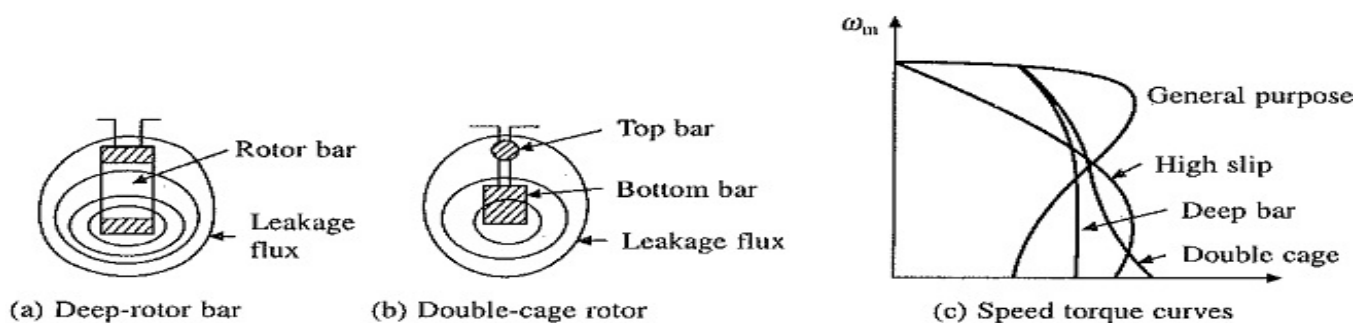


Fig. 3 Induction motors of special designs

- A general purpose induction motor is designed to operate at low slip at full load in order to have good running performance.
- Depending on the rating, full load slip varies from 2 to 7%. Such a motor has high starting current (5-8 times) and low starting torque (full load torque to twice full load torque).
- High Slip Induction Motors** operate at a large slip (between 10 and 40% at full load) they are called high slip motors. High slip motors are also suitable for fan drives where speed is controlled by stator voltage control and are found among both—squirrel-cage and wound rotor

Industrial Drives and Applications-BEE702

- In squirrel-case induction motors, good starting performance (low starting current and high starting torque) is realized without appreciably affecting full load performance by the use of deep-bar rotor or double-cage rotor motors.
- Rotor frequency changes from 50 Hz to 1-3 Hz as the speed changes from standstill to full load: Variation of rotor frequency is utilized in these motors to vary rotor resistance from a large value at standstill to a very small value at full speed.
- Thus, while starting and low speed performance is improved, full load performance is not appreciably effected.

Operations with Unbalanced Source Voltages and Single phasing

- Supply voltage may sometimes become unbalanced. Further, motor terminal voltage may be unbalanced
- A three-phase Unbalanced Source Voltages: (V_a , V_b and V_c) can be resolved into set of three-phase balanced positive sequence (V_p), negative sequence (V_n) and zero sequence (V_0) voltages, using symmetrical component relations.

$$\begin{aligned} V_p &= \frac{1}{3} (V_a + \alpha V_b + \alpha^2 V_c) \\ V_n &= \frac{1}{3} (V_a + \alpha^2 V_b + \alpha V_c) \\ V_0 &= \frac{1}{3} (V_a + V_b + V_c) \end{aligned}$$

- Motor performance can be calculated for positive and negative sequence voltages separately. Zero sequence line voltage is zero due to absence of neutral connection (assumed).
- Positive sequence voltages produce an air-gap flux wave which rotates at synchronous speed in the forward direction.

- For a forward rotor speed ω_m , slip s is given by

$$s = \frac{\omega_{ms} - \omega_m}{\omega_{ms}}$$

- For positive sequence voltages, equivalent circuits are same, except that V is replaced by V_p .
- The positive sequence rotor current and torque are obtained by replacing V by V_p , in Eqs. (4) and (10). Thus

$$I'_{rp} = \frac{V_p}{\left(R_s + \frac{R'_r}{s} \right) + j(X_s + X'_r)}$$

Industrial Drives and Applications-BEE702

- Interaction between positive sequence air-gap flux wave and positive sequence rotor currents produce positive sequence torque T_p

$$T_p = \frac{3}{\omega_{ms}} \left[\frac{V_p^2 R'_r / s}{\left(R_s + \frac{R'_r}{s} \right)^2 + (X_s + X'_r)^2} \right]$$

- Negative sequence voltages produce an air-gap flux wave which rotates at synchronous speed in the reverse direction. The slip is

$$s_n = \frac{-\omega_{ms} - \omega_m}{-\omega_{ms}}$$

$$s_n = (2 - s)$$

Negative sequence produce a flux opposite to positive sequence

- Equivalent circuits of Fig. 1 are applicable when s is replaced by $(2 - s)$ or s_n , and V are replaced by V_n
- Expressions for rotor current and torque

$$I'_m = \frac{V_n}{\left(R_s + \frac{R'_r}{2 - s} \right) + j(X_s + X'_r)}$$

$$T_n = -\frac{3}{\omega_{ms}} \left[\frac{V_n^2 R'_r / (2 - s)}{\left(R_s + \frac{R'_r}{(2 - s)} \right)^2 + (X_s + X'_r)^2} \right]$$

- Negative sequence torque T_n is produced due to interaction between negative sequence flux wave and negative sequence rotor currents
- The rms rotor current and torque are given by

$$I'_r = (I'^2_{rp} + I'^2_{rn})^{1/2} \quad (21)$$

$$T = T_p + T_n$$

$$= \frac{3}{\omega_{ms}} \left[\frac{V_p^2 R'_r / s}{\left(R_s + \frac{R'_r}{s} \right)^2 + (X_s + X'_r)^2} - \frac{V_n^2 R'_r / (2 - s)}{\left(R_s + \frac{R'_r}{2 - s} \right)^2 + (X_s + X'_r)^2} \right] \quad (22)$$

- Positive sequence, negative sequence and the resultant speed-torque characteristics are shown in Fig. 4(a)

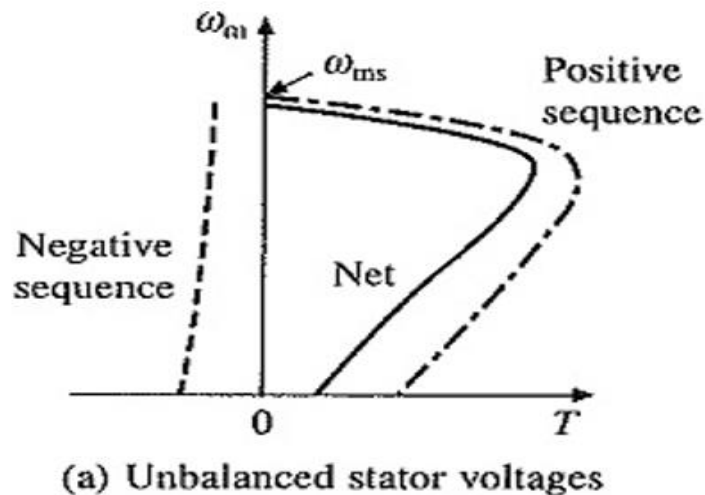
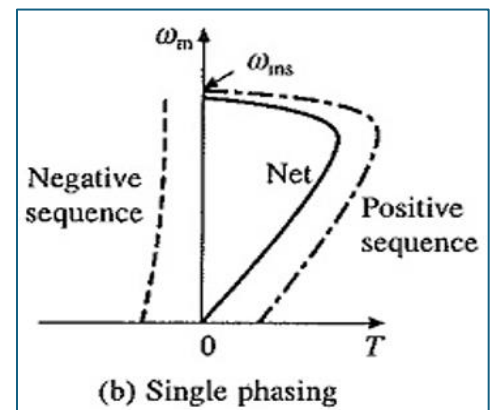


Fig. 4 Speed-torque curves of an induction motor with unbalance stator voltages

Single phasing

- Single phasing (when supply to any one phase fails) is the extreme case of unbalancing, when $V_p = V_n$. At zero speed, s is also equal to s_n , consequently starting torque is zero. Speed-torque curves for single phasing are shown in Fig. 4(b)
- positive sequence torque T_p is produced by interaction between positive sequence air-gap flux wave and positive sequence rotor currents
- Negative sequence torque T_n is produced due to interaction between negative sequence flux wave and negative sequence rotor currents.
- Torques are also produced due to interactions between positive sequence flux wave and negative sequence rotor currents, and negative sequence flux wave and positive sequence rotor currents
- However, these torques are pulsating in nature with zero average values.
- The pulsating torques cause vibrations which reduce the life of motor and produce hum.
- Equations I_r' and T indicates that while the torque is reduced, copper losses (and also core losses) are increased.
- Thus, the Unbalanced Source Voltages Operations reduces the motor torque capability and efficiency.
- when the unbalance in voltages is more than 5% - Motor is not allowed to run for a prolonged period, to prevent burning of the motor



Industrial Drives and Applications-BEE702

- For the same reason, motor is disconnected from the source whenever single phasing occurs, unless the single phasing is always accompanied by a light load.

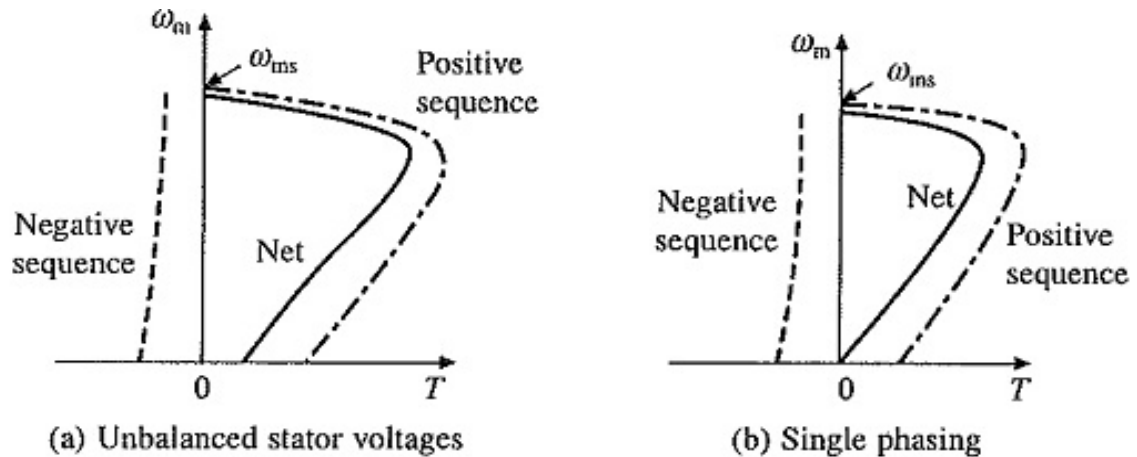
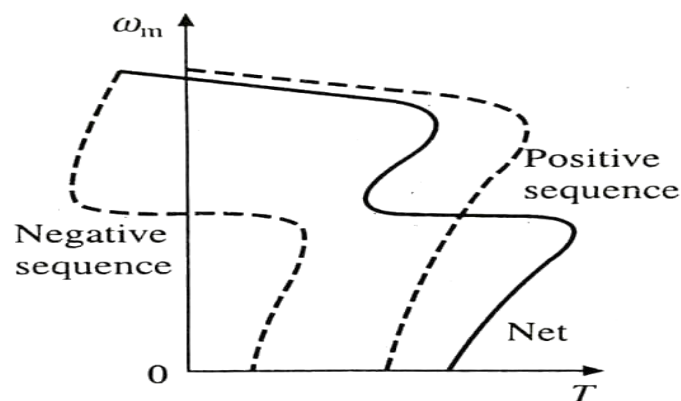


Fig. 4 Speed-torque curves of an induction motor with unbalance stator voltages

Operation with Unbalanced Rotor Impedances

- Unbalanced rotor impedance causes unbalance in rotor currents.
- The unbalanced rotor currents can be resolved into positive and negative sequence components.
- Positive sequence rotor currents produce driving torque similarly as of balanced rotor resistances.
- The negative sequence components produce a rotating field which moves with respect to rotor at a speed $(-S\omega_{ms})$ and in space at a speed of $\omega_s(1-2S)$
- Interaction between positive and negative sequence components produces pulsating torques with zero average values.
- These sequence components reduces the motor torque, but copper and core losses are increased there by reducing efficiency.
- When single phasing occurs in the rotor, peak value of steady-state voltage is excess of twice normal.
- If one phase of the stator also gets opened at the same time, so that the voltages becomes higher.



Speed-torque curves with unbalanced rotor resistance

Analysis of Induction Motor Fed From Non-Sinusoidal Voltage Supply

- When fed from an Semiconductor Converters (inverter or Cycloconverter), the motor terminal voltage is non-sinusoidal but it has half-wave symmetry.
- A non-sinusoidal waveform can be resolved into fundamental and harmonic components using Fourier analysis. **Because of half-wave symmetry only odd harmonics will be present.**
- The Harmonic can be divided into positive sequence, negative sequence and zero sequence
- The harmonics, which have the same phase sequence as that of fundamental are called positive sequence harmonics.
- The Harmonic having phase sequence opposite to fundamental are called negative sequence harmonics.
- The harmonics which have all three-phase voltages in phase are called zero sequence harmonics.
- The fundamental phase voltage components : *with the phase sequence ABC.*

$$\begin{aligned}V_{AN} &= V_1 \sin \omega t, \\V_{BN} &= V_1 \sin (\omega t - 2\pi/3) \\V_{CN} &= V_1 \sin (\omega t - 4\pi/3)\end{aligned}$$

- The corresponding 5th and 7th harmonic phase voltages are

$V_{AN} = V_5 \sin 5\omega t$	$Ps \text{ shift}=0$
$V_{BN} = V_5 \sin 5(\omega t - 2\pi/3) = V_5 \sin(5\omega t - 4\pi/3)$	$Ps \text{ shift}=240$
$V_{CN} = V_5 \sin 5(\omega t - 4\pi/3) = V_5 \sin(5\omega t - 2\pi/3)$	$Ps=120$

The 5th harmonic has a phase sequence ACB, hence it is a negative sequence harmonic

$V_{AN} = V_7 \sin 7 \omega t$	$Ps \text{ shift}=0$
$V_{BN} = V_7 \sin 7(\omega t - 2\pi/3) = V_7 \sin(7\omega t - 2\pi/3)$	$Ps \text{ shift}=120$
$V_{CN} = V_7 \sin 7(\omega t - 4\pi/3) = V_7 \sin(7\omega t - 4\pi/3)$	$Ps=240$

7th harmonic has the phase sequence ABC, which is the same as that of fundamental.

Hence it is a positive sequence harmonic

- A positive sequence harmonic ‘m’ will produce a rotating field, moves in the same direction as the fundamental at a speed m times that of the fundamental field.
- Negative sequence harmonic ‘m’ will produce rotating field, moves in opposite direction to the fundamental at m times its speed.

- Zero sequence components do not produce a rotating field.
- For fundamental component, the equivalent circuits of Fig.1 will be applicable.

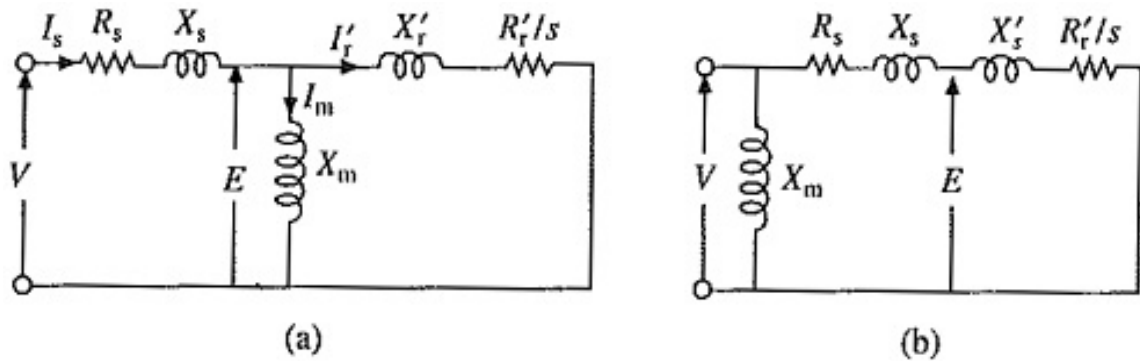


Fig. 1 Per-phase stator referred equivalent circuits of an induction motor

- For any 'm'th harmonic, equivalent circuit is shown in Fig. 2. Each reactance has been increased by a factor 'm'. Due to the skin effect resistances will also be increased several times.

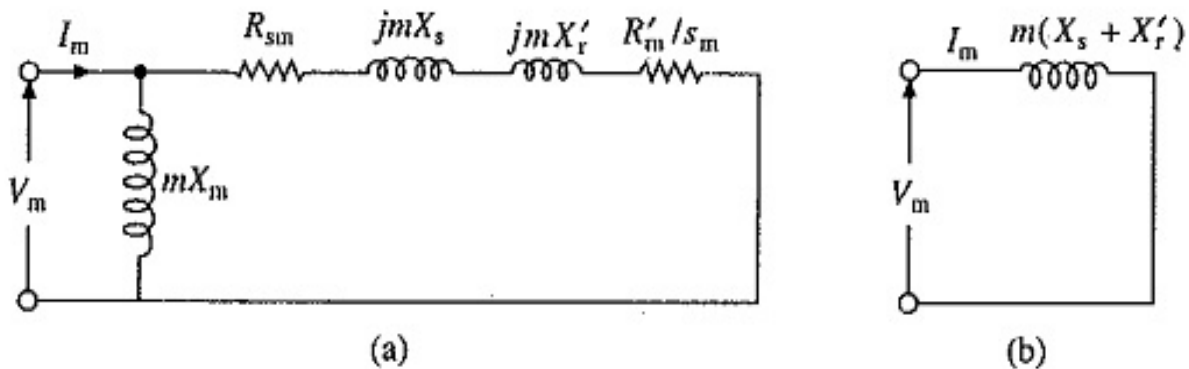


Fig. 2 Harmonic equivalent circuits of an induction motor

Slip s_m for the m^{th} harmonic is given by:

$$s_m = \frac{m\omega_{ms} \mp \omega_m}{m\omega_{ms}}$$

*Negative sign -harmonics produce forward rotating fields
positive sign - produce backward rotating fields.*

- Since s_m is close to unity, resistance $((R_r/s_m))$ has a small value. Simplified Eq't ckt is shown in Fig 2b.
- When fed from a semiconductor converter, it can be shown that the net torque produced by harmonics is close to zero.
- The motor torque can be evaluated from equivalent circuits of Fig. 2(b):

$$T = \frac{3}{\omega_{ms}} \left[\frac{V^2 R_r'/s}{\left(R_s + \frac{R_r'}{s} \right)^2 + (X_s + X_r')^2} \right]$$

- Fundamental component of rotor current:

$$\bar{I}'_r = \frac{V}{\left(R_s + \frac{R'_r}{s}\right) + j(X_s + X'_r)}$$

- The m^{th} harmonic current is calculated from Fig. 2(b) as

$$I_m = \frac{V_m}{mX}$$

- When stator is star-connected triplen harmonics (third harmonic and its multiples) will not flow. The rms motor current I_{rms} will then be

$$I_{\text{rms}}^2 = I_s^2 + \sum_{m=5,7,11,\dots} I_m^2$$

- When motor is delta-connected, triplen harmonics will circulate in delta, but will not flow in the source.
- The source current therefore can be obtained by multiplying I_{rms} given by above Eq. by $\sqrt{3}$. The rms motor phase current will be obtained by

$$I_{\text{rms}}^2 = I_s^2 + \sum_{m=3,5,\dots} I_m^2$$

Effects of Induction Motor Fed From Non-Sinusoidal Voltage Supply

- For a given motor torque and power, rms current flowing through the motor has a higher value.
- Due to skin effect, harmonic rotor resistance has higher value.
- Harmonics increase the copper loss, core losses, thereby reducing efficiency. Hence motor has to be derated.
- Harmonics produces pulsating torques due to interaction between the rotating field produced by one harmonic and rotor current of another harmonic. Harmonic 5, 7, 11 and 13 are major contributors of torque pulsations.
- When motor supply frequency is not very low, the frequency of torque pulsations is large enough to be filtered out by motor inertia.
- The torque pulsations do not have significant effect on motor speed, although they do increase noise and reduce motor life due to vibrations.
- However, when motor supply frequency is low, these torque pulsations cause pulsations in speed.
- The motor then does not move smoothly but have jerky motion.

Starting of Induction Motor Drives

Starting of IM drives are required to have following features:

- Motor should develop enough starting torque to overcome friction, load torque and inertia of motor-load system, and thus, complete the starting process within a prescribed time limit.
- Starting current magnitude should be such that it does not cause the overheating of the machine and does not cause a dip in the source voltage beyond a permissible value.
- Usually, a motor draws 5 to 7 times rated current during starting.
- When load torque during starting and motor-load-inertia are not large, motor can always be started direct on line.
- For small size motors voltage dip in the supply line is usually below acceptable level.
- When the motor is of large capacity and/or fed from a weak system, some starting arrangement becomes necessary for reducing the starting current.
- When either the load torque during starting is high or load inertia is large, the Starting of Induction Motor Drives process takes long time.
- If motor carries large current during starting, it will get damaged due to overheating. Therefore, motor cannot be started direct on line.
- In these cases, those methods of starting which allow a decrease in starting current without a decrease in starting torque are employed.
- In some applications an increase in starting torque accompanied by a decrease in starting current may be required.
- In a squirrel-cage motor some measures for improvement of starting performance may be taken at design stage, as in case of high slip, deep-bar and double cage squirrel-cage motors.

Types of Starters

1. **Star-delta starter**
2. **Auto-transformer starter**
3. **Reactor starter**
4. **Saturable reactor starter**
5. **Part winding starter**
6. **AC voltage controller starter**
7. **Rotor resistance starter is used for starting of wound-rotor motor**

Starting of Induction Motor

1. Star-Delta Starter:

- In this method, an induction motor designed to operate normally with delta connection is connected in star during starting.
- This allows reduction in stator voltage and current by $1/\sqrt{3}$.
- Since motor torque is proportional to the square of stator terminal voltage, starting torque is reduced to one-third.
- A circuit for star-delta starting is shown. Circuit breakers CB_m and CB_s are closed to start the machine with star connection.
- When steady-state speed is reached CB_s is opened and CB_r is closed to connect machine in delta.

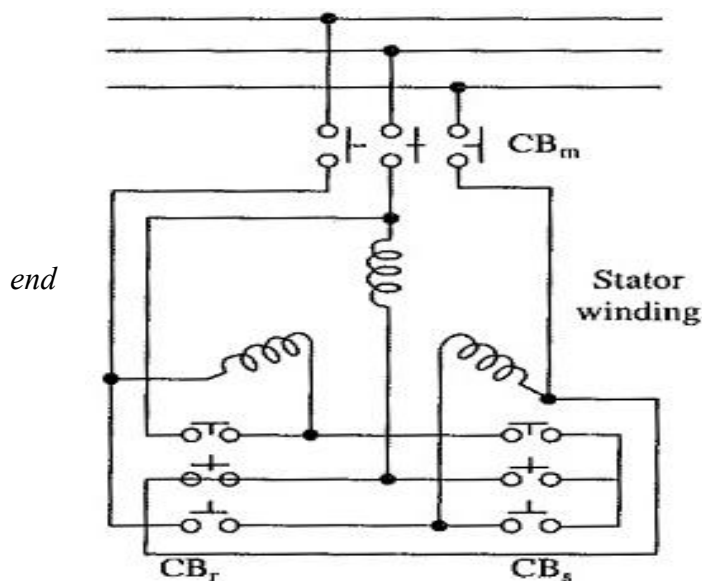


Fig. Start-delta starting

Note:

CB_m- Supply switch

CB_s- closed: Stator RYB terminals are shorted forming Star connection.

CB_r- closed: When Motor speed reaches 80%, forming Delta connection (for running)

2. Auto-transformer Starter:

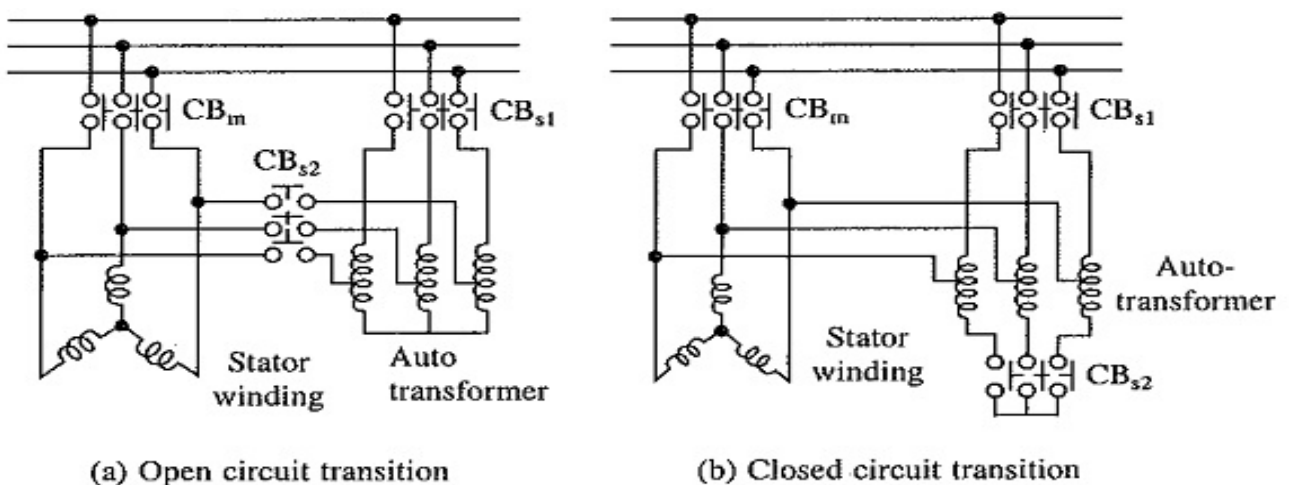
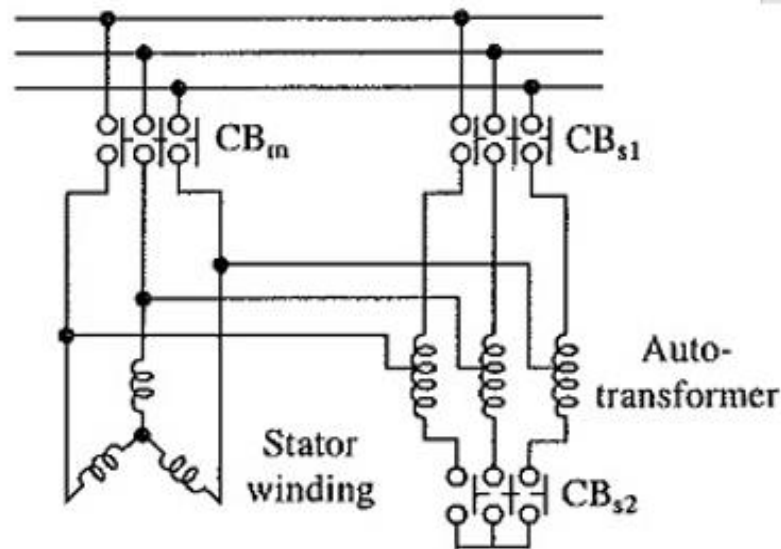


Fig. Auto-transformer starting

- Reduced voltage for starting is obtained from an auto-transformer.
- For a secondary to primary turns ratio of a_T , motor terminal voltage and stator current are reduced by a_T . This reduces the current drawn from supply by a_T^2 .
- Since torque is proportional to the square of motor terminal voltage, it is also reduced by a_T^2 . After the motor has accelerated, it is connected to full supply voltage.
- An auto-transformer starter circuit, with open circuit transition is shown in Fig.(a). First, CB_{s1} is closed followed by CB_{s2} . When motor has accelerated to full speed, CB_{s2} is opened and CB_m closed. Now CB_{s1} is opened to disconnect auto-transformer from switch.

3. Closed-circuit transition



(b) Closed circuit transition

- A large current inrush is produced at the time of reconnection when induced and supply voltages are out of phase. When the current inrush is not acceptable, closed circuit transition is employed.
- A closed-circuit transition scheme for an auto-transformer starter is shown
- It employs three circuit breakers: CB_{s1} , CB_{s2} and CB_m . First CB_{s2} is closed to close the star point connection of the auto-transformer. CB_{s1} is closed next. This completes low voltage connection of auto-transformer and the motor starts.
- After steady-state speed is reached, circuit breaker CB_{s2} is opened. Motor now runs with the upper part of auto-transformer phase windings in series with the stator. Windings simply function as series reactors. Now circuit breaker CB_m is closed, which bypasses series reactors and connects motor directly to the supply.

4. Reactor Starter

- Starting current can be reduced by connecting a three-phase reactor in series with stator.
- When motor reaches full speed, the reactor is bypassed.
- Here, CB_m is closed to start the machine.
- After full speed is reached, CB_s is closed to short the reactor.
- It is advantageous to connect reactor at the neutral end of stator winding.
- This minimizes its voltage rating and also maintains its voltage and the voltage of breaker CB_s at neutral potential during normal motor operation

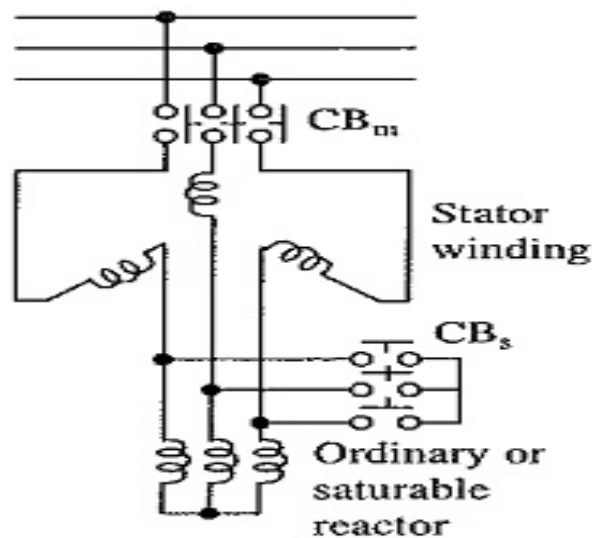


Fig. Reactor starting

5. Soft Start Using Saturable Reactor

- In some applications, starting torque must be controlled steplessly.
- For example in textile machines, it must be varied smoothly, otherwise fibre threads will break during starting. Such a starting arrangement is termed **Soft Start**.
- Thyristor voltage controller scheme is now widely used for soft start.
- A number of existing drives also employ saturable reactor starter is a three-phase saturable reactor is connected in series with the stator.
- Saturable reactor has dc control winding.
- Reactance of saturable reactor can be varied steplessly by changing the control winding current.
- For starting, reactance is initially set at the highest value.
- Starting torque is close to zero.
- Reactance is now reduced smoothly by increasing the control winding current.

Industrial Drives and Applications-BEE702

- This gives stepless variation of starting torque.
- Consequently, motor starts without any jerk and accelerates smoothly.

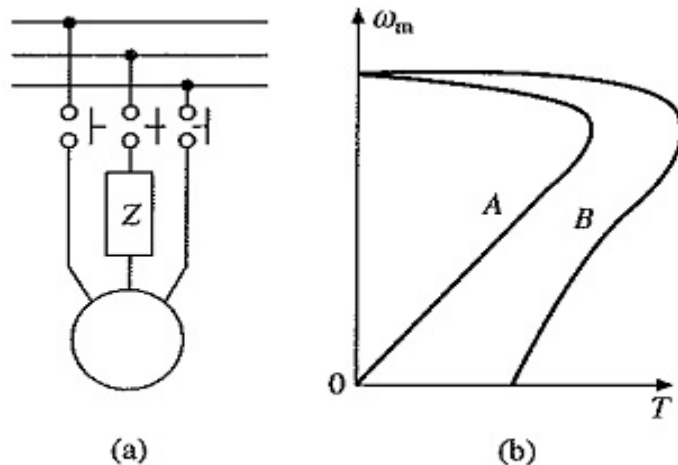


Fig. Starting with a single variable impedance in the stator

6. Part Winding Starting

- Some squirrel-cage motors have two or more stator windings which are connected in parallel during normal operation.
- During starting, only one winding is connected.
- This increase stator impedance and reduces starting current.
- Such a starting scheme is called **Part Winding Starting**.
- Its implementation for a machine with two stator windings is shown in Fig
- Machine starts with winding 1 when CB_m is closed.
- After full speed is reached, CB_s is closed to connect winding 2.

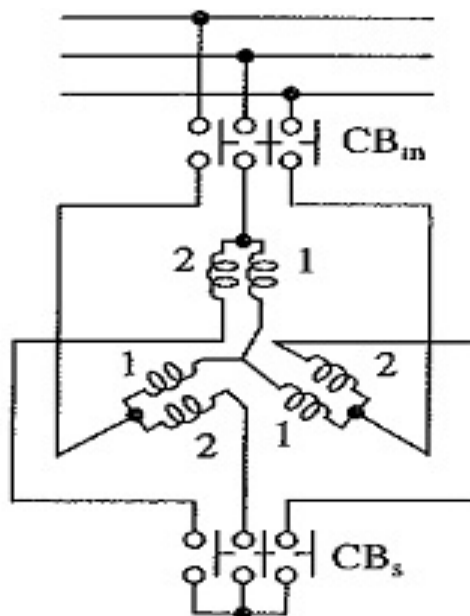


Fig. Part winding starting

7. Rotor Resistance Starter:

- Wound-rotor motors are generally started by connecting external resistors in the rotor circuit (Fig. (a)).
- The highest value of resistance is chosen to limit current at zero speed within the safe value.
- As the motor accelerates, sections in the external resistor are cut out one-by-one by closing contacts.
- As the motor accelerates, sections in the external resistor are cut out one-by-one by closing contacts C_1 , C_2 and C_3 so as to limit the rotor current between specified maximum and minimum values Fig b.
- Here, the starting torque and torque-to-current ratio are high.
- It is, therefore, suitable for applications requiring fast acceleration, frequent starts and stops, starting with heavy load, and starting with high inertia load.
- While maximum torque is independent of rotor resistance value, speed at which maximum torque is produced can be controlled by changing the value of external resistors, External resistors can therefore be varied to accelerate the machine at maximum torque.

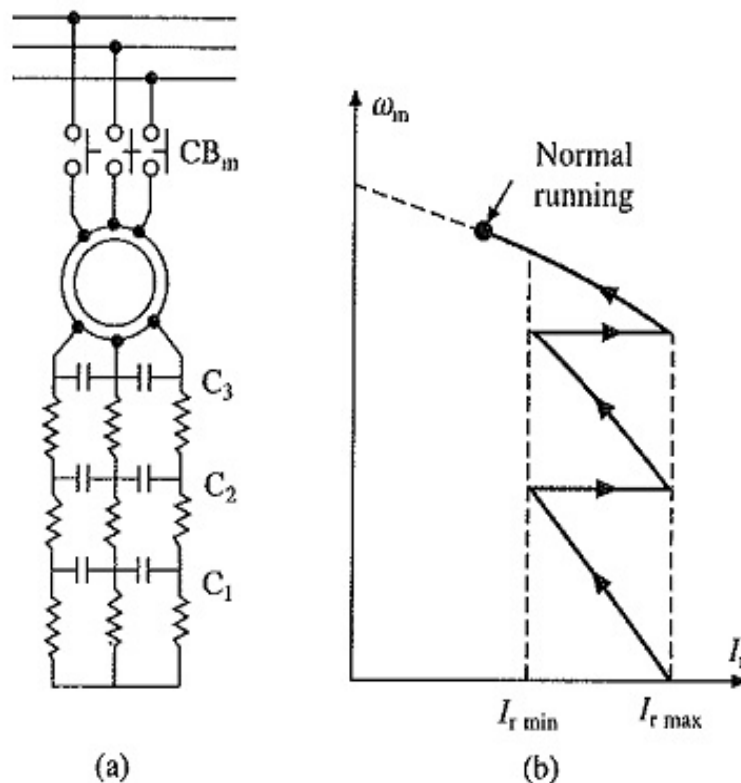


Fig. Rotor resistance starting

Braking of Induction Motor

Following methods are employed for Braking of Induction Motor Drive:

1. **Regenerative braking**
2. **Plugging or reverse voltage braking**
3. **Dynamic (or Rheostatic) braking further categorized as:**
 - i. AC dynamic braking
 - ii. Self-excited braking using capacitors
 - iii. DC dynamic braking
 - iv. Zero sequence braking

1. Regenerative Braking:

The power input to an induction motor is given by $P_{in} = 3VI_s \cos \phi_s$

- For motoring operation: $\Phi_s < 90^\circ$
- If $N > N_s$, The rotor speed becomes greater than synchronous speed, relative speed between the rotor conductors and air-gap rotating field reverses.
- This reverses the rotor induced emf, rotor current and component of stator current which balances the rotor ampere turns.
- Consequently, angle Φ_s becomes greater than 90° and power flow reverses, giving **regenerative braking**.
- Equations (1)-(13) are applicable, except that slip is negative.
- The nature of speed-torque characteristic is shown in Fig.
- When fed from a source of fixed frequency, regenerative braking is possible only for speeds greater than synchronous speed
- With a variable frequency source it can also be obtained for speeds below synchronous speed.
- Main advantage of regenerative braking is that generated power is usefully employed.
- Main drawback being that when fed from a constant frequency source, it cannot be employed below synchronous speed.

$$s = \frac{\omega_{ms} - \omega_m}{\omega_{ms}} \quad (1)$$

$$T_{max} = \frac{3}{2\omega_{ms}} \left[\frac{V^2}{R_s \pm \sqrt{R_s^2 + (X_s + X_r')^2}} \right] \quad (13)$$

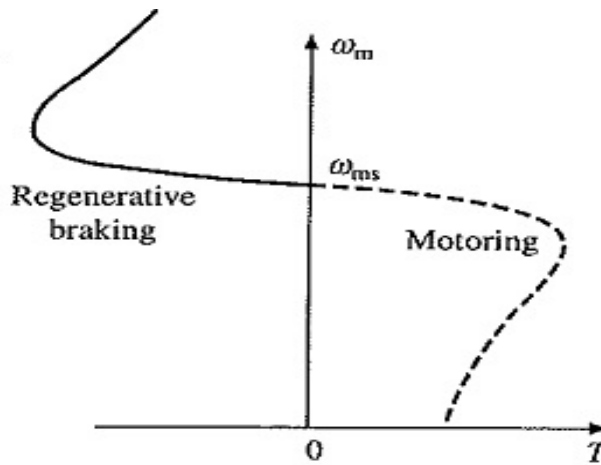


Fig. Regenerative braking

2. Plugging or Reverse Voltage Braking

- When phase sequence of supply of the motor running at a speed is reversed, by interchanging connections of any two phases of stator with respect to supply terminals, **operation shifts from motoring to plugging** as shown in below Fig..
- Plugging characteristics** are actually extension of motoring characteristics for negative phase sequence from quadrant III to II.
- Reversal of phase sequence reverses the direction of rotating field.
- If the slip for plugging is denoted by s_n , the
$$s_n = \frac{-\omega_{ms} - \omega_m}{-\omega_{ms}} = 2 - s$$
- Motor performance can be calculated from Eqs. (4)-(10) when s is replaced by s_n or $(2 - s)$.
- Since at the instant of switchover to plugging, slip can be upto 2, the rotor induced voltage can be twice of its value at zero speed.
- Consequently, motor current is large, although braking torque is low.
- A special case of plugging occurs when an induction motor connected to positive sequence voltages is driven by an active load in the reverse direction (quadrant IV).
- Crane hoist is one such application where large rotor resistance is employed shown in fig.b

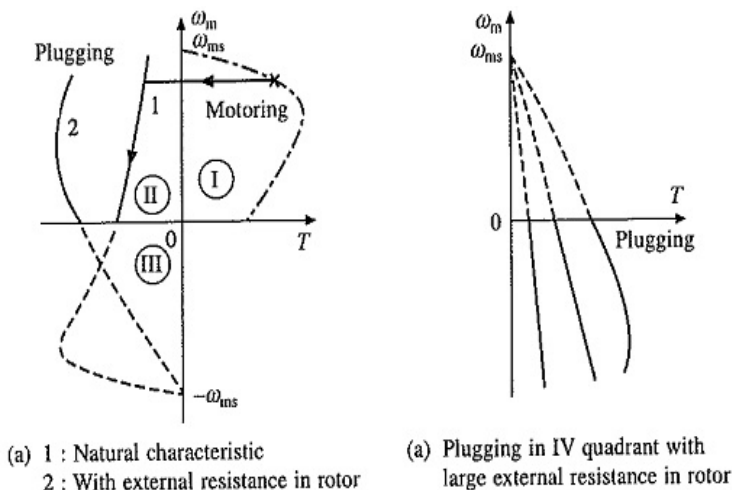


Fig. Plugging

$$\bar{I}'_r = \frac{V}{\left(R_s + \frac{R'_r}{s}\right) + j(X_s + X'_r)} \quad (4)$$

$$T = \frac{3}{\omega_{ms}} \left[\frac{V^2 R'_{r/s}}{\left(R_s + \frac{R'_r}{s}\right)^2 + (X_s + X'_r)^2} \right] \quad (10)$$

3. Dynamic Braking of Induction Motor(or Rheostatic Braking)

a. AC Dynamic Braking

AC Dynamic Braking of Induction Motor is obtained when the motor is run on a single phase supply by disconnecting one phase from the source and either leaving it open (Fig.b) or connecting it with another machine phase (Fig.c). The two connections of Figs. (b) and (c) are, respectively, known as two and three lead connections.

- When connected to a 1-phase supply, the motor can be considered to be fed by positive and negative sequence three-phase set of voltages.
- Net torque produced by the machine is sum of torques due to positive and negative sequence voltages.
- When rotor has a high resistance, the net torque is negative and braking operation is obtained.

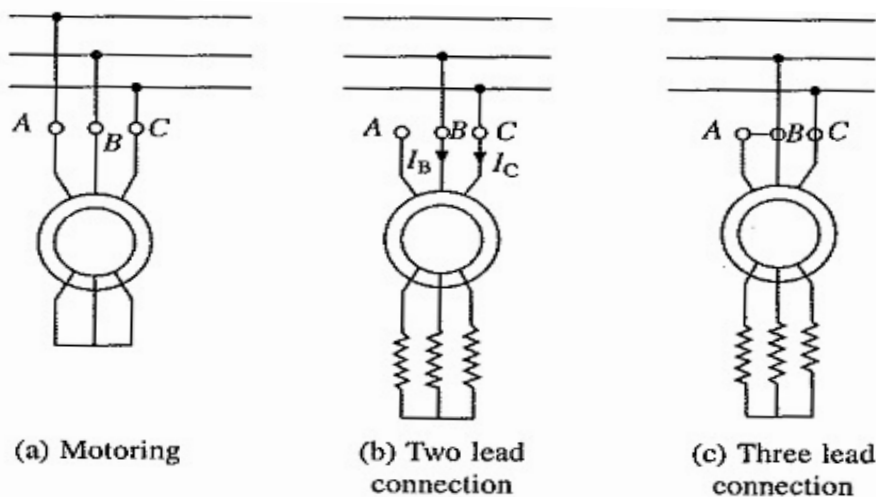


Fig. ac dynamic braking of a wound rotor motor

Two Lead Connection:

Two Lead Connection: Assume that phase A of a Y-connected motor is open circuited. Then $I_A = 0$ and $I_C = -I_B$.

- Hence positive and negative sequence components I_p and I_n , respectively.

$$\bar{I}_p = \frac{1}{3} (\bar{I}_A + \alpha \bar{I}_B + \alpha^2 \bar{I}_C) = \frac{1}{3} (0 + \alpha \bar{I}_B - \alpha^2 \bar{I}_B) = j \bar{I}_B / \sqrt{3}$$

$$\bar{I}_n = \frac{1}{3} (\bar{I}_A + \alpha^2 \bar{I}_B + \alpha \bar{I}_C) = \frac{1}{\sqrt{3}} (0 + \alpha^2 \bar{I}_B - \alpha \bar{I}_B) = -j \bar{I}_B / \sqrt{3}$$

As positive and negative sequence components are equal and opposite, two equivalent circuits can be connected in series opposition. Voltage to be applied to this series combination will be

$$\begin{aligned} (\bar{V}_p - \bar{V}_n) &= \frac{1}{3} (\bar{V}_A + \alpha \bar{V}_B + \alpha^2 \bar{V}_C) - \frac{1}{3} (\bar{V}_A + \alpha^2 \bar{V}_B + \alpha \bar{V}_C) \\ &= \frac{1}{3} (\alpha - \alpha^2) (\bar{V}_B - \bar{V}_C) = \frac{1}{3} (j\sqrt{3}) (\bar{V}_{BC}) = j \bar{V}_{BC} / \sqrt{3} \end{aligned}$$

- The nature of speed-torque curves for positive and negative sequence currents, and net torque are shown in Fig. (b)

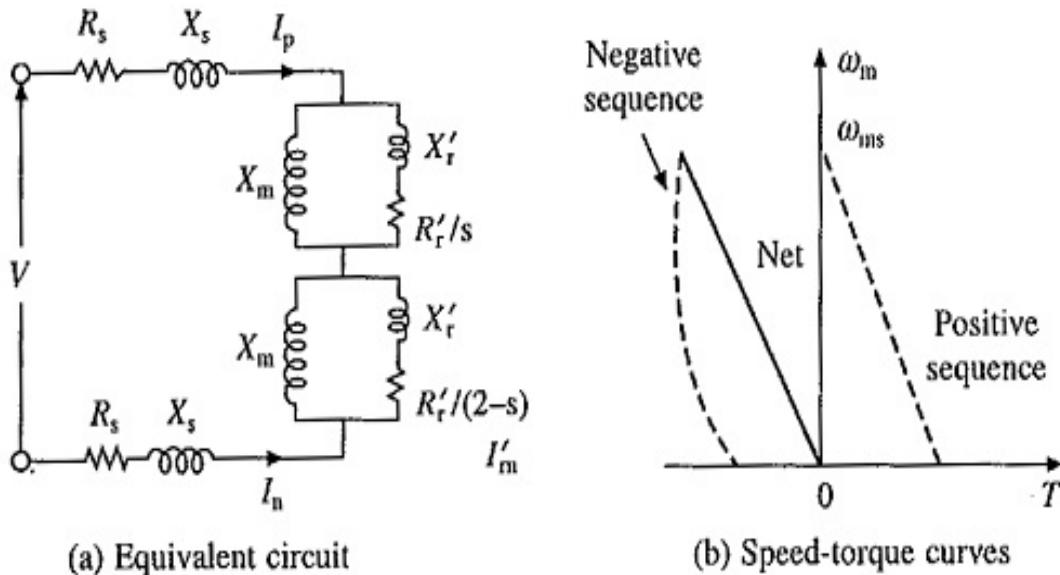


Fig. ac dynamic braking with two lead connection

Three Lead Connection:

Three Lead Connection: Here two phases of Y-connected motor winding are connected in parallel in series with the third phase (Fig. (c)). Let phases A and B be connected together, then

$$\bar{V}_{AB} = 0, \bar{V}_{BC} = \sqrt{3} V \quad \text{and} \quad \bar{V}_{CA} = -\sqrt{3} V$$

$$\bar{V}_p (\text{line}) = (\bar{V}_{AB} + \alpha \bar{V}_{BC} + \alpha^2 \bar{V}_{CA})/3$$

$$= (0 + \alpha \sqrt{3} V - \alpha^2 \sqrt{3} V)/3 = jV$$

$$\bar{V}_n (\text{line}) = (\bar{V}_{AB} + \alpha^2 \bar{V}_{BC} + \alpha \bar{V}_{CA})/3$$

$$= (0 + \alpha^2 \sqrt{3} V - \alpha \sqrt{3} V)/3 = -jV$$

$$V_p (\text{phase}) = V_n (\text{phase}) = \frac{V}{\sqrt{3}}$$

- In contrast to two lead connection, here magnitude of positive and negative sequence components of voltage are equal and not the positive and negative sequence components of currents.

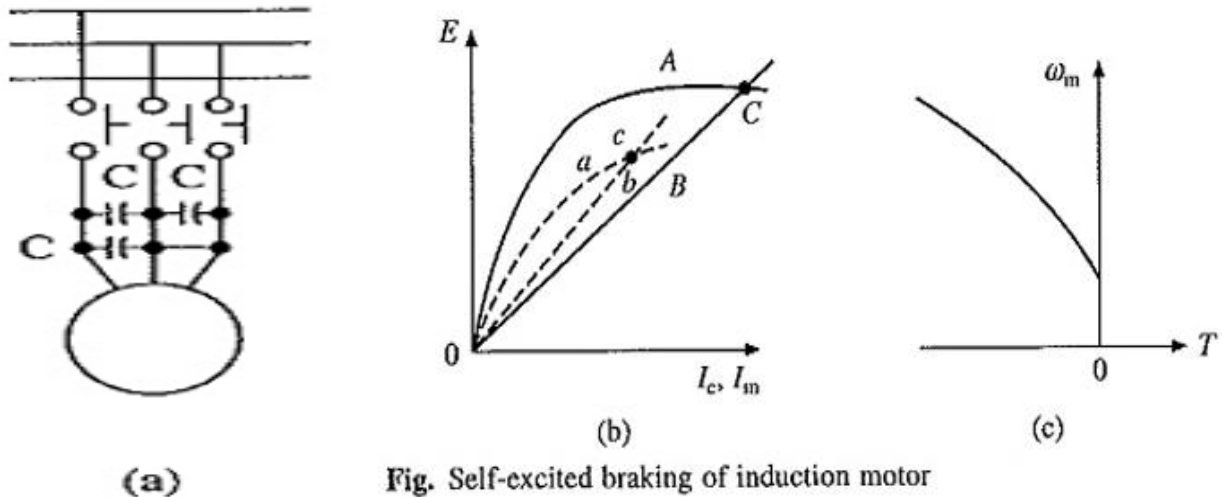
b. Self-Excited Braking Using Capacitors

- In this method three capacitors are kept permanently connected across the motor terminals.

Industrial Drives and Applications-BEE702

- Values of capacitors is so chosen that when disconnected from the line, motor works as a self-excited induction generator.
- Braking connection is shown in Fig. (a) and self-excitation process is explained in Fig. (b) for no load condition.
- Curve A is no load magnetization curve of the machine at a given speed, and line B represents the current through capacitors, given by

$$I_c = \sqrt{3} E / X_c = \sqrt{3} E \omega C$$



c. DC Dynamic Braking

- It is obtained when the stator of an induction motor running at a speed is connected to a dc supply.
- Two commonly used connections, two and three lead, for star and delta connected stators are shown in below Fig.1
- A method of getting dc supply with the help of a diode bridge for two lead connection is shown in Fig. 2
- DC current flowing through the stator produces a stationary magnetic field. Motion of rotor in this field induces voltage in the rotor winding.
- So, Machine works as a generator. Generated energy is dissipated in the rotor circuit resistance, thus giving Dynamic Braking of Induction Motor

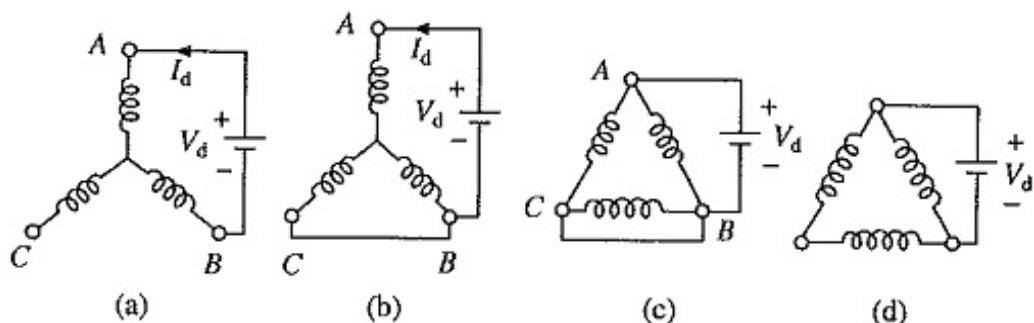


Fig. Various stator connections for dc dynamic braking. (a) and (d) are two lead connections and (b) and (c) are three lead connections

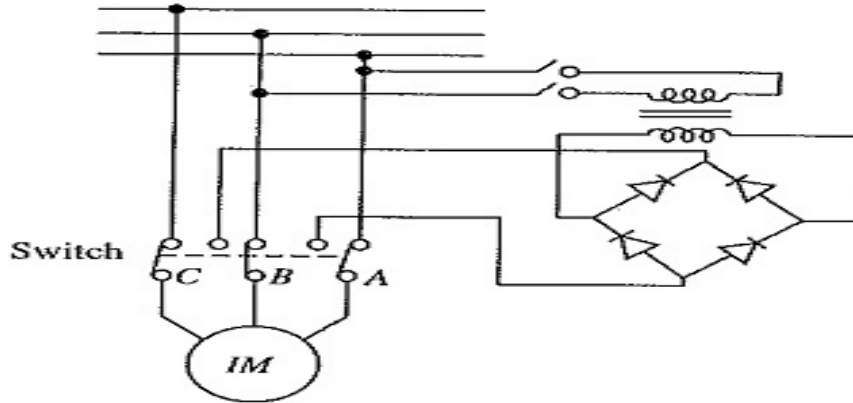


Fig. Details of two lead dc dynamic braking connection

d. Zero Sequence Braking

- In this braking, three stator phases are connected in series across either a single phase ac or a dc source as shown in Fig. (a). Such a connection is known as a Zero Sequence Connection, because currents in all the stator windings are co-phasal.
- The mmf caused zero-sequence currents produces a magnetic field having three times the number of poles for which the machine is actually wound.
- With an ac supply, resultant field is stationary in space and pulsates at the frequency of supply.
- With dc supply, resultant field is stationary in space and is of constant magnitude.
- The nature of speed-torque curves for ac and dc supply is shown in Fig. (b)
- With ac supply, braking could be used only up to one-third of synchronous speed.
- However, braking torques produced by this connection are considerably larger than motoring. Motor essentially works in regenerative braking.
- For motors with low rotor resistance, a significant part of generated energy is recovered.
- It does not require large rotor resistance, can be used both—with squirrel-cage and wound-rotor motors.

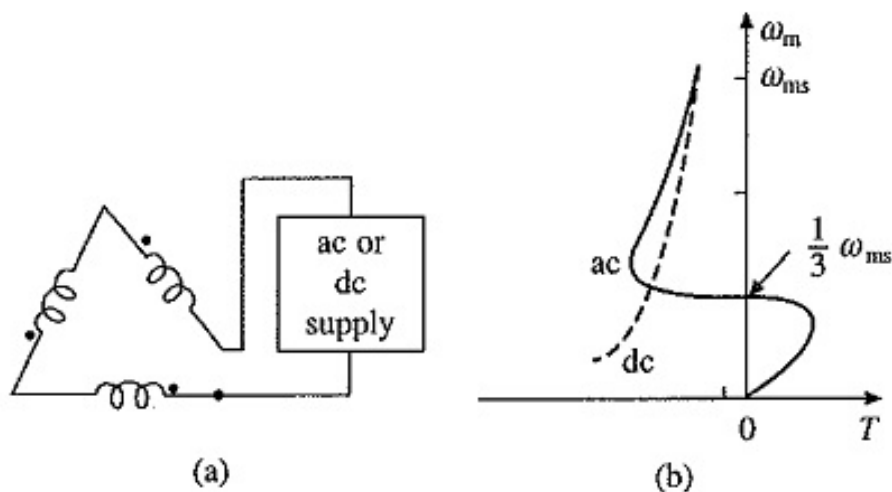


Fig. Zero-sequence braking

Transient Analysis of Induction Motor

For starting and plugging operation of machine, torque is given by Eq. $\frac{T}{T_{\max}} = \frac{2}{\frac{s}{s_m} + \frac{s_m}{s}}$

$$J = \frac{d\omega_m}{dt} = T(\omega_m) - T_l(\omega_m)$$

yields $J \frac{d\omega_m}{dt} = \frac{2T_{\max}}{s/s_m + s_m/s} - T_l(\omega_m)$

In some cases, Eq. will be in integrable form, and therefore, can be solved analytically.

It is useful to examine the transients for starting and plugging operations when operating on no load. Thus, from Eq. for no load operation

Starting and Plugging

Differentiating Eq. $\omega_m = \omega_{ms}(1 - s)$

$$\frac{d\omega_m}{dt} = -\omega_{ms} \frac{ds}{dt}$$

$$J \frac{d\omega_m}{dt} = \frac{2T_{\max}}{s/s_m + s_m/s}$$

$$dt = -\frac{\tau_m}{2} \left(\frac{s_m}{s} + \frac{s}{s_m} \right) ds$$

$$\tau_m = \frac{J\omega_{ms}}{T_{\max}}$$

τ_m is the mechanical time constant of motor. It is defined as the time taken by motor to reach its synchronous speed from standstill under constant accelerating torque equal to the maximum torque of the motor.

time required to start an induction motor on no load is $t_s = -\frac{\tau_m}{2} \int_1^{0.05} \left(\frac{s}{s_m} + \frac{s_m}{s} \right) ds$

- When operating on no load, steady-state is reached when $s = 0$. Thus during starting slip changes from 1 to 0. However, if eq is integrated for $s = 1$ to $s = 0$ an infinite value is obtained for starting time.
- When final speed is the steady-state equilibrium speed, transients are considered to be over when 95% range of speed is covered. Therefore, in Eq. integration is done from $s = 1$ to $s = 0.05$. Solving gives

$$t_s = \tau_m \left[\frac{1}{4s_m} + 1.5 s_m \right]$$

Industrial Drives and Applications-BEE702

- Thus starting time is a function of s_m . Starting time has a minimum value of $1.22\tau_m$ at $s_m = 0.4$. From Eq. ,

$$s_m = \pm \frac{R'_r}{\sqrt{R_s^2 + (X_s + X'_r)^2}}$$

- when R_s is negligible, rotor resistance required to start the motor in minimum time is

$$(R'_{rm})_s = 0.4 (X_s + X'_r)$$

- Time required for stopping by plugging, when initially running at synchronous speed, can be expressed as

$$t_b = -\frac{\tau_m}{2} \int_2^1 \left(\frac{s}{s_m} + \frac{s_m}{s} \right) ds = \tau_m \left[0.345 s_m + \frac{0.75}{s_m} \right]$$

- Stopping time is again a function of s_m . It has a minimum value of $1.027\tau_m$ at $s_m = 1.47$. Corresponding value of rotor resistance is

$$(R'_{rm})_b = 1.47 (X_s + X'_r)$$

- Time required for speed reversal by plugging when running on no load is given by

$$t_r = -\frac{\tau_m}{2} \int_2^{0.05} \left(\frac{s}{s_m} + \frac{s_m}{s} \right) ds = \tau_m \left[3.69 s_m + \frac{1}{s_m} \right]$$

- Minimum time for reversal is thus $2.88\tau_m$ and corresponding value of s_m is 0.52. Rotor resistance required for speed reversal by plugging in minimum time is

$$(R'_{rm})_r = 0.52 (X_s + X'_r)$$

Calculation of Energy Losses

Let us next obtain expressions for energy loss in motor windings for starting and plugging operations. The rotor winding loss for starting can be written as

As the machine is operating under no load

$$J \frac{d\omega_m}{dt} = T$$

$$E_{sr} = \int_0^{t_s} 3 I_r'^2 R'_r dt$$

$$E_{sr} = \int_0^{t_s} \omega_{ms} T s dt$$

$$-J\omega_{ms} \frac{ds}{dt} = T$$

$$Tdt = -J\omega_{ms}ds$$

$$E_{sr} = - \int_1^0 J\omega_{ms}^2 s ds = \frac{1}{2} J\omega_{ms}^2$$

- Rotor winding energy loss is equal to the kinetic energy stored in moving parts at completion of the starting process, and it is independent of the starting time or rotor resistance.
- Energy loss in stator winding, neglecting magnetizing current is
- Total winding loss during starting under no load is

$$E_{ss} = \int_0^{t_s} I_r'^2 R_s dt$$

$$= \frac{1}{2} J\omega_{ms}^2 \left(\frac{R_s}{R_r'} \right)$$

$$E_s = \frac{1}{2} J\omega_{ms}^2 \left(1 + \frac{R_s}{R_r'} \right)$$

- Rotor winding loss during stopping by plugging under no load can be written as

$$E_{sr} = \int_2^1 J\omega_{ms}^2 s ds = \frac{3}{2} J\omega_{ms}^2$$

- Rotor winding loss can be reduced when started by using methods based on the variation of synchronous speed.
- As an example let us consider a motor with an arrangement for doubling the pole number.
- Let it be started with higher pole number for which the synchronous speed is $\omega_{ms}/2$.
- Then, from eq, rotor copper loss for change of speed from 0 to $\omega_{ms}/2$ will be $J\omega_{ms}^2/8$.
- Now the pole number is lowered. Consequently, rotor copper loss for speed range $\omega_{ms}/2$ to ω_{ms} will be

$$E'_{sr} = \int_{0.5}^0 J\omega_{ms}^2 s ds = \frac{J\omega_{ms}^2}{8}$$

- Thus, total rotor winding loss is $J\omega_{ms}^2/4$, which is one-half of the copper loss when there is no provision for doubling the pole number.

Speed control of Induction Motor

The conventional methods of speed control of induction motors are,

Stator Side

- Stator voltage control
- Variable frequency control
- Stator current control
- V/f control
- Changing the number of poles on stator

Rotor Side

- Rotor resistance control
- Injecting emf in the rotor

Stator Voltage Control of Induction Motor

- Speed of induction motor can be varied by varying the voltage applied to the stator winding.
- Torque developed by 3 phase induction motor is directly proportional to the square of the stator voltage as given by the equation

$$T_m = \frac{3}{2\pi N_s} \times \frac{S E_2^2 R_2}{R_2^2 + (S X_2)^2} \text{-----1} \quad \longrightarrow \quad N_s = \frac{3}{2\pi T_m} \times \frac{S E_2^2 R_2}{R_2^2 + (S X_2)^2} \text{-----2}$$

$(S X_2)^2$ is very small as compared to R_2^2

$$T_m \propto \frac{S E_2^2}{R_2}$$

R_2 is constant

$$T_m \propto S E_2^2$$

E_2 is proportional to the supply voltage V_1

$$T_m \propto S V_1^2 \text{-----3}$$

Current is proportional to voltage

$$\bar{I}'_r = \frac{V}{\left(R_s + \frac{R'_r}{s} \right) + j(X_s + X'_r)}$$

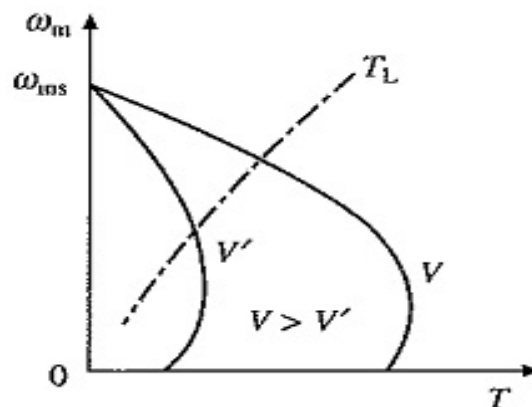
- From eqn. 2, it is clear that any reduction in supply voltage will reduce the motor speed.
- But from equation 3, it is seen that any reduction in supply voltage will reduce the torque also.

Industrial Drives and Applications-BEE702

- Therefore, as voltage is reduced to reduce speed, for the same current motor develops lower torque.
- This method is suitable for applications where torque demand reduces with speed, such as for fan and pump drives.
- Hence this method is used in applications where torque demand reduces with reduction in voltage.
- Here the slip increases at low speeds. Hence the efficiency of the drive reduces
- If stator copper loss, core loss, and friction and windage loss are ignored, then from eqns , motor efficiency η is given by

$$\eta = \frac{P_m}{P_g} = (1 - s)$$

- The equation shows that the efficiency falls with decrease in speed
- The speed control is essentially obtained by dissipating a portion of rotor input power in rotor resistance.
- Thus, not only the efficiency is low the power dissipation occurs in the rotor itself, which may overheat the rotor.
- Because of these reasons, this drive is employed in fan and pump drives of low power rating and for narrow speed range.

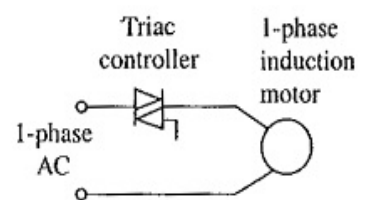


Control by AC Voltage Controllers and Soft Start

Single-phase

Domestic fan motors, which are always single-phase, are controlled by a single-phase Triac Voltage Controller shown in Fig. a

- Speed control is obtained by varying firing angle of the Triac.
- These controllers, commonly known as solid state fan regulators, are now preferred over conventional variable resistance regulators because of higher efficiency.

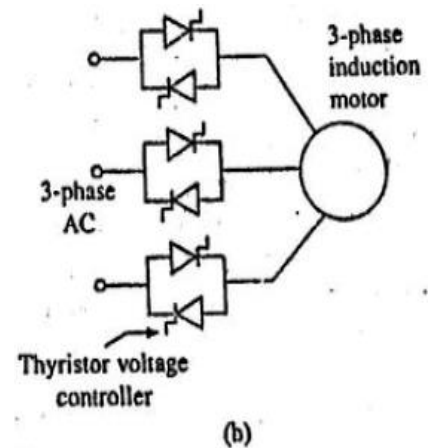


(a)

Three-phase motors.

- Industrial fans and pumps are usually driven by Three-phase motors.
- Fig.b shows thyristor voltage controller for speed control of 3-phase motors.
- Motor may be connected in star or delta. In delta connection, third harmonic voltage produced by motor back emf causes circulating current through the windings which increases losses and thermal loading of motor.
- Speed control is obtained by varying conduction period of thyristors.
- For low power ratings, anti-parallelled thyristor pair in each phase can be replaced by a triac.

Fig. Stator voltage control by semiconductor voltage Controller



Variable Frequency Control From Voltage sources

- i. Variable Frequency Control of an Induction Motor
- ii. Slip Speed Control
- iii. Torque and Power Limitations, and Modes of Operation

i. Variable Frequency Control of an Induction Motor

- The motor speed can be controlled by varying supply frequency.
- Voltage induced in stator is proportional to the product of supply frequency and air-gap flux.
- Increase in flux will saturate the motor, which increases the magnetizing current, distort the line current and voltage, increase the core loss and the stator copper loss, and produce a high-pitch acoustic noise.
- Increase in flux beyond the rated value is undesirable from the consideration of saturation effects, a decrease in flux is also avoided to retain the torque capability of the motor.
- Therefore, the variable frequency control below the rated frequency is carried out at rated air-gap flux by varying terminal voltage with frequency so as to maintain (V/f) ratio constant at the rated value.
- From Torque Equation

$$T_{\max} = \frac{3}{2\omega_{ms}} \left[\frac{V^2}{R_s \pm \sqrt{R_s^2 + (X_s + X_r')^2}} \right]$$

Industrial Drives and Applications-BEE702

- The maximum torque in an induction motor is given by,

$$T_{max} = \frac{K(V/f)^2}{\frac{R_s}{f} \pm \sqrt{\left(\frac{R_s}{f}\right)^2 + 4\pi^2(L_s + L_r')^2}}$$

K is a constant and L_s & L_r' are the stator and stator referred rotor inductances.

+ve: For Motoring

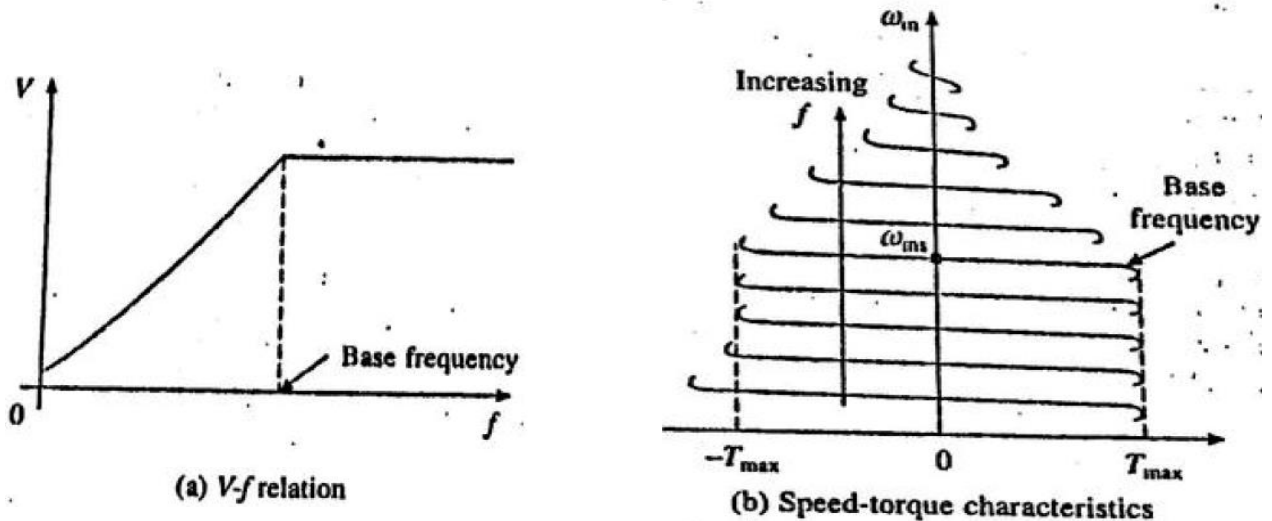
-ve: Braking

- At high frequencies, the value of (R_s/f) will be very much less than $2\pi(L_s + L_r')$.
- So (R_s / f) can be neglected and hence the torque equation becomes

$$T_{max} = \pm \frac{K(V/f)^2}{\sqrt{4\pi^2(L_s + L_r')^2}}$$

$$T_{max} = \pm \frac{K(V/f)^2}{2\pi(L_s + L_r')}$$

- From above equation, if the ratio (V / f) is kept constant, the motor can produce a constant maximum torque, T_{max} . i.e constant torque operation.
- Hence if maximum torque needs to be maintained constant at low speeds, then (V / f) ratio must be increased.
- Near to base speed (or rated speed), the supply voltage will be maximum and it cannot be increased further.
- Therefore, above base speed, the frequency is changed by keeping supply voltage constant.
- But this will decrease the maximum torque produced by the motor as per the equation
- Variation in terminal voltage with frequency is shown in Fig.a.
- (V/f) ratio is increased at low frequency to keep maximum torque constant, which leads to increase in maximum braking torque.
- Above base speed, frequency is changed with V kept constant.
- up to base frequency- (V/f) ratio is kept constant.
- above base frequency- V is kept constant and frequency is varied
- At low frequencies - (V/f) ratio is Increased to keep maximum torque constant.
- Corresponding speed torque curves are shown in Fig. 6.33(b) both for motoring and braking operations.
- The curves suggest that speed control and braking operation are available from nearly zero speed to above synchronous speed.



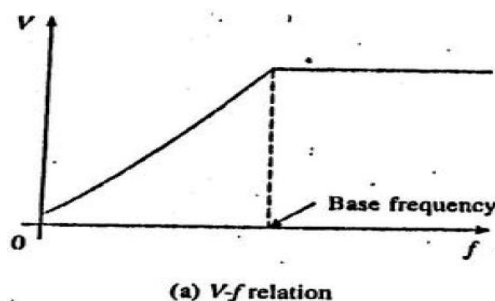
The variable frequency control provides good running and transient performance because of the following features:

- (a) Speed control and braking operation are available from zero speed to above base speed.
- (b) During transients (starting, braking and speed reversal) the operation can be carried out at the maximum torque with reduced current giving good dynamic response.
- (c) Copper losses are low, and efficiency and power factor are high as the operation is restricted between synchronous speed and maximum torque point at all frequencies.
- (d) Drop in speed from no load to full load is small.

Variable Frequency Variable Voltage Source (VFVS).

Block diagram of variable frequency speed control scheme is shown in Fig. The motor is fed from a variable frequency variable voltage source (VFVS).

- V^* and f^* are voltage and frequency commands for VFVS.
- Flux control block produces a voltage command V^* for VFVS in order to maintain the relation between V^* and f^* as per the v-f Graph.
- Reference frequency f^* is changed to control speed.
- A delay circuit is introduced between f^* and f' , so that even when f^* is changed by a large amount, f^* will change only slowly so that motor speed can track changes in f^* .
- Thus restricting the motor operation for each frequency between synchronous speed and the maximum torque point.
- VFVS can be a voltage source inverter or a cycloconverter.



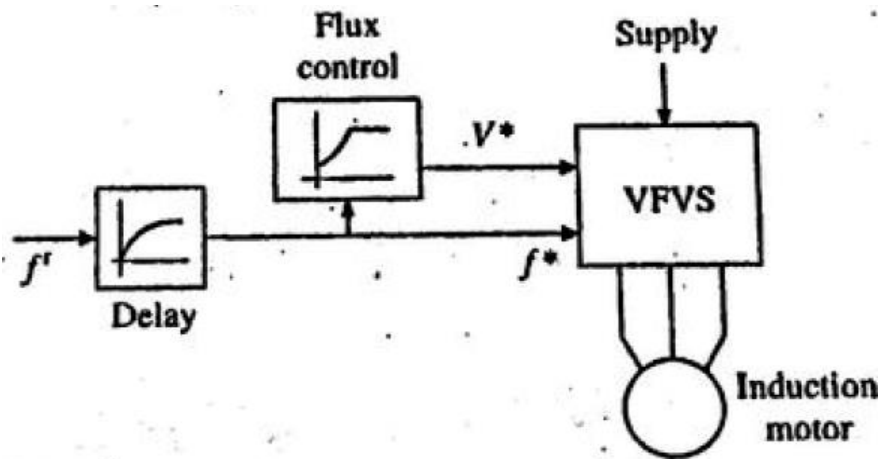


Fig. 6.34 Variable frequency control

ii. Slip Speed Control

- Let V and f be the rated voltage and frequency of the machine.
- When the motor is operated below the base speed with constant (V/f) control, for a frequency kf , the terminal voltage will be kV , where k is a factor such that, $0 \leq k \leq 1$.
- Thus, as frequency is changed from 0 to f , k changes from 0 to 1 and voltage changes from 0 to V .

$$I_r = \frac{V}{\sqrt{(R'_r/ks)^2 + (X_s + X'_r)^2}} \quad T = \frac{3}{\omega_{ms}} \left[\frac{V^2 R'_r / ks}{(R'_r/ks)^2 + (X_s + X'_r)^2} \right]$$

- If (ks) is maintained constant as k is varied, then rotor current I'_r and torque T will remain constant.
- Since the slip is small I'_r will be in phase with voltage. Since flux is constant I_m will also be constant. Now

$$I_s = \sqrt{I'^2_r + I_m^2} = \text{constant}$$

- Thus if the motor operation is carried out at constant value of ks as the frequency is varied then the motor will operate at a constant current and torque.
- Let us examine the meaning of ks .
- At frequency kf ,

$$\text{Synchronous speed} = k\omega_{ms}$$

$$\text{Slip } s = \frac{k\omega_{ms} - \omega_m}{k\omega_{ms}}$$

$$ks = \frac{k\omega_{ms} - \omega_m}{\omega_{ms}} = \frac{\omega_{sl}}{\omega_{ms}}$$

$$\omega_{sl} = k\omega_{ms} - \omega_m$$

$$\omega_{sl} = k\omega_{ms} - \omega_m$$

Where ω_{sl} is the slip speed, which is the difference in the rotating field speed $k\omega_{ms}$ and rotor speed ω_m .

- It is also the drop in motor speed from its no load speed, when the machine is loaded.
- Operation of the machine at a constant slip speed also implies the operation at a constant rotor frequency as shown below

$$ks = \frac{(kf)s}{f} = \frac{f_r}{f} = \frac{\omega_r}{\omega}$$

where f_r and ω_r are rotor frequency in Hz and rad/sec, respectively.

For $s < s_m$, $(R'_r/ks) \gg (X_s + X'_r)$,

$$T = \frac{3}{\omega_{ms}} \left[\frac{V^2 R'_r / ks}{(R'_r / ks)^2 + (X_s + X'_r)^2} \right]$$



$$T = \frac{3V^2}{R'_r \omega_{ms}} (ks) = \text{constant} \cdot \omega_{sl}$$

For $s < s_m$, the speed torque curves are nearly straight lines. Since they are also parallel, the speed-torque curves are approximately parallel straight lines for $s < s_m$.

- Let us next consider the operation above base speed. As stated earlier, machine operates at a constant voltage V. Now

$$I'_r = \frac{V}{\sqrt{\left(R_s + \frac{R'_r}{s}\right)^2 + k^2(X_s + X'_r)^2}}$$

- As the frequency is higher than the rated $k > 1$. Since the operation is again constrained between the synchronous speed and the maximum torque, slip has a small value, hence

$$I'_r = \frac{sV}{R'_r} = \frac{V}{R'_r} \left(\frac{k\omega_{ms} - \omega_m}{k\omega_{ms}} \right)$$

$$(k\omega_{ms} - \omega_m) = \omega_{sl} = \frac{R'_r}{V} \omega_{ms} (kI'_r)$$

- Since the slip is small, I'_r is in phase with V. If the machine copper loss is neglected, the developed power P_m is given by

$$P_m = 3VI'_r$$

iii. Torque and Power Limitations, and Modes of Operation

- When the stator current has the maximum permissible value, these will represent the maximum torque and power capabilities of the motor in Variable Frequency Control of Induction Motor Drive.
- Zero to base speed ω_{mb} : The motor has a constant maximum torque, hence the drive operates in constant torque mode.
- Beyond the speed ω_{mc} : The machine is operated at a constant slip speed and the maximum permissible current and maximum power are allowed to decrease.
- Now the motor current reduces inversely with speed and torque decreases inversely as the speed squared.
- The operation in this region is required in drives requiring wide speed range but low torque at high speeds.
- For example in traction applications the drive operates in this region when running at full speed because the torque required in steady state at high speeds is very small compared to its value during acceleration.

